Proportionality in two dimensions: resolving an old dilemma of political (mis)representation

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Abstract

The contemporary idea that a parliament acts as a mirror of voter's party preferences is historically still very young. At the dawn of modern representative government, parliamentary elections were held within small, territorially confined communities. Legislators appeared as agents of their local electorates. While local representation implies a multitude of small districts, the realization of proportional party representation on the other hand requires a small quantity of large districts. Given the apparent incompatibility of the two concepts, representative democracies are faced with the challenge of finding a middle ground between the two antipodes. Through the interaction of legal coercion and horizontal transfer, eight of the Swiss cantons moved to a new, so-called biproportional apportionment method, which reconciles the two antithetical concepts. The new method constitutes a remarkable innovation both, theoretically and methodically. While several electoral researchers devoted themselves to studying the methodical side of the new system, the community has so far failed to engage into a discussion of its contributions on a theoretical level. The paper aims at closing this gap by putting the biproportional method in a context of the history of representation theory and by outlining its undervalued innovations that become particularly effective in a heterogeneous federal state such as Switzerland.

Keywords: biproportional apportionment, political representation, Swiss cantons

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1. Introduction

In 1968, Giovanni Sartori argued that "citizens in modern democracies are represented *by* and *through* parties" (1968: 471). Today, barely anyone would question the validity of his statement. It is a commonplace that in the context of Western democracies, the main channel through which political representation is realized is through parties and their adherents. There is, however, a second dimension of political representation that has receded into the background but still effectively shapes electoral processes today: territorial representation.

In most contemporary electoral systems, the electoral region is territorially subdivided into smaller districts. The idea behind the subdivision is to bring the representatives closer to the electors (Ramírez et al. 2008). However, dividing the electoral region into smaller districts has a drawback with major implications. The decentralization of elections creates electoral barriers for parties what lowers the permeability of the electoral system. Unless a party reaches a certain minimal share of votes, it will return empty-handed (even if it was entitled to a seat *in globo*). This well-studied mechanical effect of electoral systems is known to suppress small parties and, consequently, their electorates. The smaller the district magnitudes, the stronger the reductive effect on the party system and the higher the discrepancies in vote-seat shares (Duverger 1954, Rae 1967, Sartori 1976, Taagepera and Shugart 1989, Lijphart 1994).

As a result, there is a conflict of goals between ensuring undistorted party representation on the one hand (by setting low hurdles) and satisfying the desire of local populations for territorial representation on the other hand (resulting in high hurdles). Different strategies have evolved to solve this teleological conflict. In some parliamentary elections, as for the German Bundestag, the New Zealand House of Representatives or in parts of Eastern Europe, proportional and majority voting elements have been combined to account for both dimensions (Shugart and Wattenberg 2001, Bochsler 2010). The most recent attempt to reconcile these antithetical concepts of representation was conducted in Switzerland at the subnational level. As a world premiere in 2006, the canton of Zurich switched to the so-called biproportional apportionment method. Since then, seven more cantons followed suit. The biproportional method has been designed to ensure equitable overall representation along both dimensions: political and geographical divisions in the population (Pukelsheim 2017).

In contrast to prior attempts to solve the dilemma, the "Swiss way" offers a methodical solution that requires only minor changes in the electoral system. Although the methodical side of the novel method has been studied in-depth already, a comprehensive theoretical framework of this pioneering innovation is missing so far. This neglect makes it difficult,

if not impossible, to fully understand the merits of the new method and its contribution to existing concepts of political representation. In this paper, I fill this gap by linking the technical side of biproportionality to its underlying theoretical concepts. First, I argue that biproportionality provides a radically new approach to solve a dilemma endogenous to electoral representation. Secondly, I argue that its potentials unfold particularly in a heterogeneous polity like Switzerland where the federalist principle of territoriality is a main pillar of state organization.

The paper is structured as follows: In chapter 2, I take a step back and trace the trajectories of the two key principles of political representation inherent to biproportionality. Chapter 3 provides a detailed description of the mechanisms of the biproportional method and shows how it innovates other proportional apportionment formulae. In chapter 4, I discuss the potentials and limitations before arriving at a conclusion in chapter 5.

2. The simultaneity of two rivalling concepts of political representation

Contemporary theoretical thinking on political representation is strongly influenced by the notion that parties act as the main link between the public and the political sphere (Pitkin 1967, Manin 1997, Rehfeld 2005, Dalton et al. 2011, Golosov 2017). The centrality of parties for the establishment and functioning of democratic institutions has been emphasized by a variety of political theorists: Max Weber referred to parties as "the children of democracy" (1990: 35), John Aldrich described them as "endemic to democracy" (1995: 3) and for Richard Katz, party government is "a synonym for representative democracy" (1987: 2). However, the contemporary notion that parties are the main vehicles for formal political representation is historically quite young. Before the emergence of political parties in modern democracies, individual legislators served as linkages between their local communities and representative institutions. The purpose of the following sections is to give a brief overview over both dimensions of political representation in electoral systems, starting with the territorial dimension (2.1), proceeding with the party dimension (2.2) and arriving at the paradox in the last section (2.3).

2.1 Constituency representation as the point of departure for representative government

Before the advent of parties in modern mass democracies, individual legislators located in small territorial constituencies served as primary linkages between the public and political decision makers (Dalton 1985: 268-271, Pukelsheim 2017). The principle of territoriality, which found expression in personalized elections within politically insulated single-member constituencies, formed the conceptional origin of modern representative government (Urbinati and Warren 2008: 389). Democracy was organized and conceived as a local event, and the voters were represented by an elect from their own "natural community of interest" (Golosov 2017: 119). Grounded in this territorial understanding of political representation, a long-lasting debate has raged over the question whether representatives should act as *delegates* or as *trustees* (Pitkin 1967, Dalton 1985, Dovi 2009). A controversy sparked by Edmund Burke's famous and equally disputed pamphlet *Reflections on the Revolution in France* (1790).

As for both models, constituency representation is realized by an act of authorization by the constituents at the outset of the election. This authorization enables the representative to act "[on] behalf of, in the interest of, as the agent of, someone else" (Pitkin 1967: 113). However, the models place conflicting demands on the legislative behavior of a representative, resulting in different degrees of autonomy of action.

The delegate model of constituency representation requires legislators to mirror the expressed policy preferences of their constituents. This leaves delegates no room for autonomy from their constituents.[1] Their authority to act as representatives is limited to making the *re-presented* literally present again. Such notions of binding political allegiance found application in the Saddle period in parts of France, the United States, and beyond in the form of instructions, pledges, imperative mandates, and the practice of discretionary revocability of representatives (recall) (Manin 1997: 163-167).

The trustee model, in contrast, requires representatives to make decisions based on their own judgment and in accordance with their own views of right action. Within the limits of authorization, the trustee enjoys full autonomy even if he/she acts against the interests of the constituents. This does not mean that the representative is not held accountable for his/her actions. Voters still have the right not to reelect the representative. Apart from that, they should not be able to exert any influence on the trustee (Dovi 2009: 4-6). In Hanna Pitkin's account (1967), these two models of constituency representation should not be conceived as mutually exclusive. Instead, she suggests accepting the paradoxical nature

[1] However, it remains rather ambiguous how this congruence in policy preferences is to be achieved, given the diversity and potential incompatibility of different policy preferences held by the constituents.

of political representation and striving to safeguard both, the autonomy of the representative and the autonomy of the represented.

Regardless of which model for describing citizen-elite relations seems empirically more accurate or appears favorable from a normative stance, they both point to the territorial patterns of early representative democracy. Electoral processes took place within territorially confined spaces and individual legislators established a corridor of formal representation between the constituents and the representative institutions of the polity.

2.2 The victory parade of political parties

The national and industrial revolution at the end of the nineteenth century triggered a profound transformation of Western European politics. New political cleavages emerged, dividing society into different camps (Lipset and Rokkan 1967). At the same time, suffrage was extended to the working class and other segments of society. This dramatically changed the composition of electorates (Dalton 1985). The increasing call of these new voter segments for political recognition led to the introduction of proportional representation, which marked another cornerstone. All these episodes of modern democracy formation gave rise to the successful establishment and proliferation of mass parties.

While for a long time, political representation was guaranteed by and through local representatives, this attachment has lost importance over time and was, to a great extent, substituted by political parties (Golosov 2017: 119). Parties offer voters manifestos with distinct policy goals. Voters, in turn, opt for those parties that best fit their own policy preferences. This illustrates that the logic of electoral choice has shifted away from the direct nomination of a candidate to a comparative assessment of abstract party manifestos. Voters think of formal political representation primarily in programmatic terms.[2] Moreover, evaluations of fairness of electoral systems are usually measured by the congruence between voters and *parties* in seats, ideology, policy preferences or other indicators (Cincea 2013: 178-179). Hence, parties did not only monopolize political power; they also deeply affected how we think about politics, political representation, and political equality.

However, the territorial dimension of political representation never entirely disappeared. In fact, it still effectively shapes electoral processes today. Some countries such as the United States or the United Kingdom have never switched to proportional representation, but still adhere to FPTP plurality voting. In their candidate-centered voting systems, the principle

[2]This becomes increasingly debatable since calls for better minority inclusion grow. Such claims could encourage voting based on descriptive, non-programmatic criteria.

of territoriality still prevails. Bicameral parliamentary systems devote a separate chamber of the national legislature to the territorial dimension of political representation and other countries such as Germany, New Zealand, Croatia, Albania, or Hungary mixed their electoral systems by combining proportional with majoritarian or plurality voting elements to account for both dimensions of representation (Bochsler 2010: 86-131).

2.3 The challenge of simultaneously meeting requirements of "both worlds"

In a less conspicuous way, the principle of territoriality is also still preserved in proportional electoral systems. As for most contemporary democracies, elections are conducted at district-level – on the national as well as on the subnational level. There are only few exceptions where legislative representatives are appointed by single-tier *at-large* elections and not by two-tier *by-district* elections (for a short overview over the two allocation levels see Blais 1991: 254-255). The purpose of electoral districts is not exclusively of organizational nature. Districts also provide local populations with representation in the larger polity's legislative body. At the same time, districts enter into a conflict with the party dimension of political representation. There is extensive literature showing that electoral districts, in particular small ones, are an impediment to fair, proportional representation of parties.[3] The smaller a district (i.e., the fewer the number of seats assigned to a district), the smaller the chances of small parties to pass the electoral hurdle imposed by the district. The cumulative effect across all districts leads, in extreme cases, to considerable disadvantages for small parties and their electorates in terms of fair representation. Large, established parties, on the other hand, are the beneficiaries of such mechanically induced (and psychologically reinforced) distortions of party competition (Duverger 1954, Rae 1967, Sartori 1976, Taagepera and Shugart 1989, Lijphart 1994). However, following more radical notions of the concept of electoral equality ("one man, one vote"), voters are not merely equal, they also have the right to be represented equally. This means that their votes must be converted into seats with equal chances of success. A normative requirement that is unattainable with small districts.

To summarize, in most modern democracies, districts are the main geopolitical units for the allocation of parliamentary seats. From the perspective of political theory, districts can be understood as an embodiment of the principle of territoriality, which played a crucial role

[3]Of course, fair representation does not depend solely on district magnitudes, but also on the distribution of the votes between the parties and on features of the electoral system such as the apportionment formula, the size of the legislature and legal thresholds. Nevertheless, district magnitude has been identified to be the most potent predictor of an electoral system's capacity to generate proportional results (Rae 1967, Taagepera and Laakso 1980, Taagepera and Shugart 1989, Gallagher 1991, Carey and Hix 2011).

in the genesis of modern democracy. However, districts have proven to be an obstacle to the approximation of "perfect" party representation, so that in extreme cases a substantial proportion of the electorate is left unrepresented. There are three solutions to solve the theoretical puzzle, differing greatly in their radicality: (1) Abolish electoral districts and introduce a single constituency.[4] (2) Mix the electoral system to allow for both dimensions of political representation to come into effect. (3) Introduce an apportionment method that is designed to satisfy both dimensions. In the remainder of this paper, I will focus on the third solution, which offers a radically new approach to reconcile the two antithetical concepts of representation. The following chapter is devoted to the description of the novel apportionment method in the Swiss context.

3. The innovation of biproportional apportionment

Biproportional apportionment is an attempt to conceptionally bridge the territorial and the party dimensions of political representation. It can be understood as an extension of one-dimensional PR electoral systems which, under certain circumstances, produce results that conflict with present notions of electoral equality. The subsequent sections show in outline the context of implementation in Switzerland (3.1), the technicalities of the new method (3.2) and how it extends one-dimensional proportional apportionment methodically and conceptionally (3.3).

3.1 The context of implementation in Switzerland

In the course of a public appeal in matters of political rights, the Federal Court as the highest legal authority in Switzerland declared the election of the Council of the City of Zurich in 2002 as "unconstitutional". The court criticized the extreme differences in seats assigned to the districts. While the smallest district received two seats, the largest district polled nineteen seats. Since seats were allocated on district-level, these differences in seats translated into differences in chances both, for voters and for parties, to get a share of seats proportional to the share of votes the parties received. The court found that the current practice discriminates against small parties and their electorates. According to the court's reasoning, it is not only unacceptable if a vote has little or no weight, but by the logical reverse-conclusion also if a vote is over-weighted. To ensure equality in electoral success among all voters and parties, the court defined a minimal requirement of nine seats

[4] With the side-effect that territories/local communities are no longer represented as such in the electoral body via district-level mandates.

per district. This should ensure minimal distortions in vote-seat proportionality. This left the canton with three options: (1) Introduce a single constituency by abolishing electoral districts. (2) Perform a redistricting of the electoral area by merging districts with fewer than nine seats. Or (3) adjust the electoral system in a way that circumvents the problem of unevenly weighted votes.[5]

The canton responded to the appeal and created a new legal basis for fair representation in all legislatures in the canton. Following the advice of Friedrich Pukelsheim, a German mathematician and expert in electoral matters, the city decided to switch to a new apportionment method. Pukelsheim presented a solution that complies with the federal directive while retaining the unaltered continuance of electoral districts (Pukelsheim and Schuhmacher 2004). The refined apportionment method, which ensures equitable overall representation of parties while taking account of geographical divisions in the population, is referred to as the biproportional (or syn. double-proportional) divisor method.

The debut of the refined method took place in February 2006 in the elections of the Council of the City of Zurich and was followed by the elections of the Cantonal Council of Zurich in the same year. Since its successful application in Zurich, another seven cantons followed suit: Schaffhausen (2008), Aargau (2009), Nidwalden (2014), Zug (2014), Schwyz (2016), Valais (2017) and most recently Uri (2020). Although all these cantons have switched to the biproportional method, there are still variations with regard to the modalities of implementation. Some cantons simultaneously introduced legal thresholds while others did not and in some cantons the method was modified to account for local peculiarities.[6] Apart from the Swiss cantons, there is yet no other national or subnational electoral system in which the biproportional method applies. This renders Switzerland a unique case to study different aspects of the new method.

3.2 Technicalities

Biproportionality is as an attempt to maximize the overall representation of parties merely by changing the apportionment formula of an electoral system instead of reforming it entirely. This is achieved through a complex process of algorithmic optimization. The main idea is

[5] As a matter of fact, the problem is even more complex. The differences in vote and seat shares depend not only on district sizes but also on the number of submitted candidate lists and on the distribution of votes to these lists. A fictious example: In a district with 10 seats, a total of 25 lists are being submitted. The first 10 each receive 4.5 percent of votes; the remaining 15 lists receive an equal share of 3.67 percent of votes. In this case, the available seats would go to the first then lists, leaving the other 15 lists which poll 55 percent of votes without weight (example from OSCE 2014: 15).

[6] A compilation of the institutional characteristics of the cantonal parliaments can be found in the appendix of this document.

to split the process of seat allocation in two separate but interconnected parts. In a first step, the seats are apportioned to the parties proportionally to the votes they received. The literature refers to this step as the *upper-tier apportionment* or *super-apportionment*. More concretely, the votes for all candidates of a party are totaled over all electoral districts to yield the party votes. However, as the biproportional method applies to multi-member districts only, voters have different number of votes to cast, depending on the district size. This translates into different levels of voting power among voters of different districts. To account for this imbalance, the different scales need to be standardized (Pukelsheim 2017: 261). By shifting the first level of allocation away from the district-level to the upper-tier level, disparities between the share of votes and share of seats obtained by a party converge to minimal values. The upper-tier allocation of seats follows the *Sainte-Laguë* method.[7]

In a next step, the overall votes obtained by the parties get sub-apportioned to the districts. This nontrivial process is the centrepiece of the new method and requires further elaboration. The lower-tier or sub-apportionment must cumulatively satisfy a number of conditions:

> *(i)* The sum of seats attributed to a party in all districts exhausts its overall number of seats.
> *(ii)* The sum of seats attributed to all parties in a district meet the district size.
> *(iii)* The sum of seats attributed to all districts exhausts the overall number of party seats.
> *(iv)* Proportionality is observed among parties within a given district, and among districts within a given party.

While conditions *(i)-(iii)* relate to the general set-up of the apportionment, condition *(iv)* adds biproportionality as a substantial supplementary criterion to the equation.[8] The challenge of finding a distribution that satisfies proportionality in both dimensions is known as the biproportional apportionment problem (Ramírez et al. 2008, Maier et al. 2010, Serafini and Simeone 2012). The problem can be solved manually but instead, running a divisor-based algorithm is advised.[9] The so-called *iterative algorithm* is designed to find those list and district divisors that satisfy (or best fit) condition *(iv)*, given conditions *(i)-(iii)*.

[7] Within the family of list-based PR voting systems, Sainte-Laguë is a divisor/highest averages apportionment method with standard rounding.

[8] The listing is a synthesis from works of Serafini and Simeone 2012: 248 and Pukelsheim 2017: 263.

[9] In 2004, Friedrich Pukelsheim and his team developed a publicly accessible Java program (BAZI) to compute electoral input matrices. In 2016, a team of mathematicians at the Rosenheim University of Applied Sciences converted the program to a R-package (see here: https://www.th-rosenheim.de/wirtschaft/wirtschaftsmathematik-aktuarwissenschaften-bachelor/labor-fuer-angewandte-mathematik-und-statistik/.

As mentioned above, the results from this iterative allocation process are designed to find Pareto-optimal biproportional distributions. However, as Gassner (1991: 341) and other authors pointed out, a perfect solution for fair two-dimensional representation does not exist. The optimization process involves so many criteria that allocation conflicts cannot always be prevented. Divisor- or multiplier-based methods cannot guarantee proportionality at district-level unless there are at most two parties and at most two districts. Indeed, it occurs that the sub-apportionment produces anomalous results in certain districts. Anomalous in the sense that the ranking of the seat numbers in a district deviates from the ranking of the vote numbers (Bochsler 2005: 22-23, Maier et al. 2010: 375). Such deviations between vote and seat shares are called *discordant seat allocations* or *seat reversals* and have been studied in depth in the work of Maier, Zachariassen and Zachariasen (2010).

3.3 Innovations

Biproportionality innovates conventional PR-based electoral systems in several ways. Many of these innovations address shortcomings of one-dimensional proportional representation. The most commonly discussed problem is the distortive effect of small districts. A circumstance resulting in an overrepresentation of big parties at the expense of small ones (Taagepera and Shugart 1989, Balinski and Young 2001). This effect is reinforced by election formulae that award remainder seats to large parties, for example the widely used *D'Hondt* formula, and also by constitutionally defined electoral thresholds. Electoral systems in which such one-sided distortions coincide are suggestive of partiality. This harms the overall integrity of the system and could discourage voters from engaging in politics.

Biproportional apportionment minimizes systemic biases by shifting the first level of apportionment away from the lower tier to the upper tier. This change is elementary as it significantly reduces the threshold-effect induced by small districts. Additionally, the biproportional method employs a divisor method (Sainte-Laguë) that operates with standard rounding of the quotients instead of rounding them down. The combination of these two modifications guarantees an equitable overall representation of party electorates and, thus, constitutes a central innovation both, in normative and in methodical terms. Another innovation is the sub-apportionment. This subsequent step is the essence of the method as it conforms to both dimensions of representation: to party and to territorial divides in the population. In this respect, political systems applying biproportionality are closely related to mixed-member PR systems, which are also devised to represent a population along two dimensions but solve the allocation problem differently (Shugart and Wattenberg 2001).

The novel method extends conventional PR systems in one more aspect. As pointed out in

earlier sections, biproportional designs pursue the goal of ensuring equal representation not only of local populations but also of party electorates. Parties should thus have the same prospects of seeing their vote shares represented. Voters in turn should have the same levels of political effectiveness regardless of their vote choice (rule of equal treatment). The concept of political equality holds thus a prominent place in the theoretical framework of the new method.

One can think of electoral equality as a multi-layered concept that follows a hierarchical ordering. In total there are three gradations: Votes can be (un)equally counted, (un)equally weighted and (un)equally successful (i.e., same chance of being effective). While all existing electoral formulae realize equality in the count value of votes as a fundamental normative prerequisite of representative systems ("one man, one vote"), only PR systems advance to higher levels of electoral equality. Since the biproportional method is designed precisely to approximate vote-seat proportionality and thus ensures that only a small fraction of votes is lost in the allocation process, it is, besides mixed-member and at-large PR systems, the only apportionment method that ensures equality in electoral success (see *Table 1*).

Table 1: Representation formulae and the different layers of electoral equality.

Equality in...	... count value	... voting power	... electoral success
MR/Plurality	Yes	No	No
Single PR	Yes	Yes	No
Double PR	Yes	Yes	Yes

4. Potentials and limitations of the new method

As shown in the previous chapter, two-dimensional proportional representation innovates one-dimensional representation in several aspects. This chapter provides a condensed overview of the potentials and limitations of the new method.

As biproportionality conforms to highest standards of electoral equality and maximizes proportionality between parties' vote and seat shares, the main beneficiaries of such an electoral reform would be expected to be small parties. Particularly fringe parties with a territorially dispersed electorate, previously hampered by the mechanical constraints of district boundaries, are likely to be on the receiving end of such a reform (Bochsler 2005). In this regard, the adoption of biproportionality would enhance electoral fairness.

Opening the floodgates to small parties would at the same time increase the fragmentation of the party system. However, a functionalist understanding suggests that moderately frag-

mented party systems are preferable to highly fragmented systems (Sartori 1976). Some scholars see a danger in highly fragmented party systems because they correspond with an increase of volatility between elections. This allegedly impedes governability and poses a threat to the stability of the political system as a whole. In the context of cantonal implementations in Switzerland, most of the reforming cantons simultaneously introduced legal thresholds, either locally by district or globally across all districts. This should prevent a "rank growth" of minor parties.

Given that biproportionality preserves electoral districts as main units of seat allocation, it allows territorial communities to find electoral representation not only through parties and their candidates but also through districts. Although pressured by the party dimension of voter representation, territorial aspects play still an important role not only in structural organization of elections, but also in the formation of political identity of territorial groups (Bengtsson and Wass 2010, Mueller 2013, Deschouwer et al. 2014, Golosov 2017). In this respect, electoral districts warrant such identity ties between voters and their political biosphere. Such ties are of particular importance in federalist states where the preservation of territorial communities and their political autonomy is a *raison d'état*.

An often-discussed limitation of the new apportionment method is the complexity and opacity of the allocation mechanism (Leuzinger 2018, Arnold 2019). The seat allocation is performed by a computer software on the basis of algorithmic optimization. The number of criteria that must be fulfilled cumulatively makes it nearly impossible for "average citizens" to replicate the sub-apportionment of seats to parties within individual districts in compliance with the requirement of vote-seat-monotonicity. This drawback can be compensated by the publication of the list- and party-divisors by the electoral authorities in the aftermath of the elections (as it is done in Swiss cantons applying the biproportional method). This allows citizens to validate the results without the need to perform the calculations themselves.

Another limitation arises from the occurrence of counterintuitive results (Bochsler 2005, Balinski and Young 2001, Golosov 2017). In a biproportional electoral system consisting of single-member districts, it might occur that within districts or within a party, the number of seats attributed to parties are disproportionate to the votes they received (as already mentioned in section 3.2). This anomaly can come in two ways: (1) Within a district (usually a small one), party A receives more seats than party B, although party B has obtained more votes. (2) Within a party list, the sub-list from district X receives more seats than the sub-list from district Y, although the sub-list from district Y obtained more votes (relatively). When the strongest party in a district receives less seats than another competing party in the same district, literature also speaks of *strongest party discordant* (Maier et al. 2010).

Such paradoxical results are unpreventable in attempts to optimally solve a two-dimensional apportionment problem. From the perspective of voters in a single-member district who cast their ballots for a losing candidate with a majority of votes, such results will be difficult to understand and to accept and they might raise a doubt about the effectiveness of their vote. For Sebastian Maier and his co-authors, who have thoroughly investigated such anomalies in a study, such local and not infrequent deviations from the vote-seat-monotonicity are "(. . .) the price one has to pay to achieve proportionality over the whole electoral region" (2010: 386).

5. Conclusion

How and to what extent does biproportional apportionment innovate conventional methods of electoral representation – theoretically as well as methodically? This twofold question was at the center of this paper and was answered with reference to Switzerland, the first and yet only country that introduced the new method on the subnational level. It has been shown that the biproportional apportionment method provides a design that allows the reconciliation of two rivalling concepts of political representation: the originary understanding of political representation as the representation of territories and the more modern notion of political representation as the representation of voters along the party dimension.

A central innovation of the new method is given by its two-step mechanism of seat allocation. While the upper-tier apportionment ensures the fair distribution of seats to parties proportionate to their share of votes, the lower-tier apportionment guarantees a fair distribution of seats to districts proportionate to population figures. Election results thus comply with both dimensions of political representation.

With the modified electoral system, small parties and their electorates, previously suppressed by mechanical hurdles, now stand a better chance of having their vote shares converted into seat shares. Large, established parties on the other hand that could previously profit from such imbalances hold the short end of the stick. However, as any electoral formula, the biproportional method too does not come without any limitations. In contrast to conventional, one-dimensional apportionment methods, the biproportional method in all its details is rather hard to comprehend for laypersons. Moreover, in certain cases, the biproportional method produces anomalous, counterintuitive results that can compromise the integrity of the electoral system. In conclusion, however, biproportional apportionment is a highly potent method when the overarching goal of elections is to produce fair outcomes along two dimensions. For this unique characteristic, the refined method has long been underestimated.

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7. Appendix

Table 2: Overview of cantonal electoral systems.

Canton	Electoral System	Apportionment Method	Legal Quorum
AI	Majoritarian	-	-
AR	Mixed	Hagenbach-Bischoff	-
AG	Proportional	Biproportional	5% local / 3% global
BS	Mixed	Sainte-Laguë	-
BL	Proportional	Hagenbach-Bischoff	-
BE	Proportional	Hagenbach-Bischoff	-
FR	Proportional	Hagenbach-Bischoff	-
GE	Proportional	Hagenbach-Bischoff	7% global
GL	Proportional	Sainte-Laguë	-
GR	Majoritarian	-	-
JU	Proportional	Hagenbach-Bischoff	-
LU	Proportional	Hagenbach-Bischoff	-
NE	Proportional	Hagenbach-Bischoff	3% global
NW	Proportional	Biproportional	-
OW	Proportional	Hagenbach-Bischoff	-
SZ	Proportional	Biproportional	1% global
SH	Proportional	Biproportional	-
SO	Proportional	Hagenbach-Bischoff	-
SG	Proportional	Hagenbach-Bischoff	-
TI	Proportional	Hare-Niemeyer	-
TG	Proportional	Hagenbach-Bischoff	-
UR	Proportional	Biproportional	-
VD	Proportional	Hare-Niemeyer	5% local
VS	Proportional	Biproportional	8% local
ZG	Proportional	Biproportional	5% local / 3% global
ZH	Proportional	Biproportional	5% local